Geotextiles and Geomembranes xxx (2013) 1–5

Contents lists available at SciVerse ScienceDirect

Geotextiles and Geomembranes

journal homepage: www.elsevier.com/locate/geotexmem

Technical note

A practical methodology for the determination of failure envelopes of fiber-reinforced cemented sands

Nilo Cesar Consoli*, Bernardo Scapini Consoli, Lucas Festugato

Department of Civil Engineering, Federal University of Rio Grande do Sul, Av. Osvaldo Aranha 99, 3 andar, Porto Alegre, RS 90035-190, Brazil

A R T I C L E I N F O

Article history:
Received 23 November 2012
Received in revised form
4 July 2013
Accepted 11 July 2013
Available online xxx

Keywords:
Fiber-reinforcement
Sand
Cement
Mohr–Coulomb failure envelope
Unconfined compressive strength
Splitting tensile strength

A B S T R A C T

This study aims to estimate the Mohr–Coulomb failure envelope of fiber-reinforced and non-reinforced artificially cemented sands based on splitting tensile strength (σ_t) and unconfined compressive strength (σ_c) of such materials, without the necessity of carrying out triaxial testing. Based on the concept previously established by Consoli et al. that the σ_t/σ_c relationship is unique for each specific soil, fiber and cement agent, it is shown that the effective angle of shearing resistance of a given fiber-reinforced or non-reinforced cemented sandy soil (ϕ') is dependent of the σ_t/σ_c ratio of such geomaterials and that effective cohesion intercept (c') is a direct function of the unconfined compressive strength (σ_c) [or splitting tensile strength (σ_t)] and σ_t/σ_c ratio of the fiber-reinforced/non-reinforced improved soil. Finally, the concepts presented herein are successfully checked for glass fiber-reinforced/non-reinforced silty sand treated with ordinary Portland cement, considering weak, moderate and strong cementation levels.

1. Introduction

Determination of Mohr–Coulomb failure envelope parameters of fiber-reinforced/non-reinforced artificially cemented soils requires carrying out triaxial tests (e.g., Clough et al., 1981; Consoli et al., 2007, 2009, 2012a, 2013; Dalla Rosa et al., 2008), simple shear (Festugato et al., 2013), amongst many other complex and time consuming tests.

An alternative methodology to estimate Mohr–Coulomb failure envelope parameters of fiber-reinforced/non-reinforced artificially cemented soils is suggested in present work. The concept is to carry out basic tests, such as unconfined compression and splitting tensile tests, whose equipment (loading machine and proving rings) can be found even under slight laboratory facilities. Besides, the methodology to be presented herein intends to allow increasing reliability and widening range of validity of the results, once the setup of basic (splitting tensile and unconfined compression) tests carried out for a given sandy soil and a specific cement agent will allow determining c' and ϕ' for any specific condition comprised inside the range of porosity and amount of cement employed during basic testing. Types of applications could fit improvement behavior of shallow foundations bearing on soil layers enhanced with cement and fiber (Consoli et al., 2003) and enhanced uplift performance of anchor plates embedded in fiber-reinforced cement stabilized backfill (Consoli et al., 2012b). During design considerations, once major difficulties usually occur during mixture procedures, the precision obtained using the methodology proposed to estimate Mohr–Coulomb failure envelope parameters of fiber-reinforced/non-reinforced artificially cemented materials is usually good.

2. Mohr–Coulomb failure theory

The Mohr–Coulomb failure theory is represented in the shear strength (τ) versus effective normal stress (σ') space by plotting Mohr semi-circles representing stress states at failure and then drawing a tangent to these semi-circles, which represents the Mohr–Coulomb failure envelope. As presented in Fig. 1a, in the Mohr–Coulomb failure theory, the shear strength (τ) of a given material is assumed, considering effective stress conditions, to vary linearly with effective normal stress (σ'), according to two parameters: effective cohesion intercept (c') and effective angle of shearing resistance (ϕ'), as shown in Eq. (1).

$$\tau = c' + \sigma' \tan \phi' \quad (1)$$

Using unconfined compression and splitting tensile tests principal stress states at failure, in which, the minimum effective

* Corresponding author.
E-mail addresses: consoli@ufrgs.br (N.C. Consoli), beconsoli@hotmail.com (B.S. Consoli), lucas@ufrgs.br (L. Festugato).

0266-1144/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.geotexmem.2013.07.010

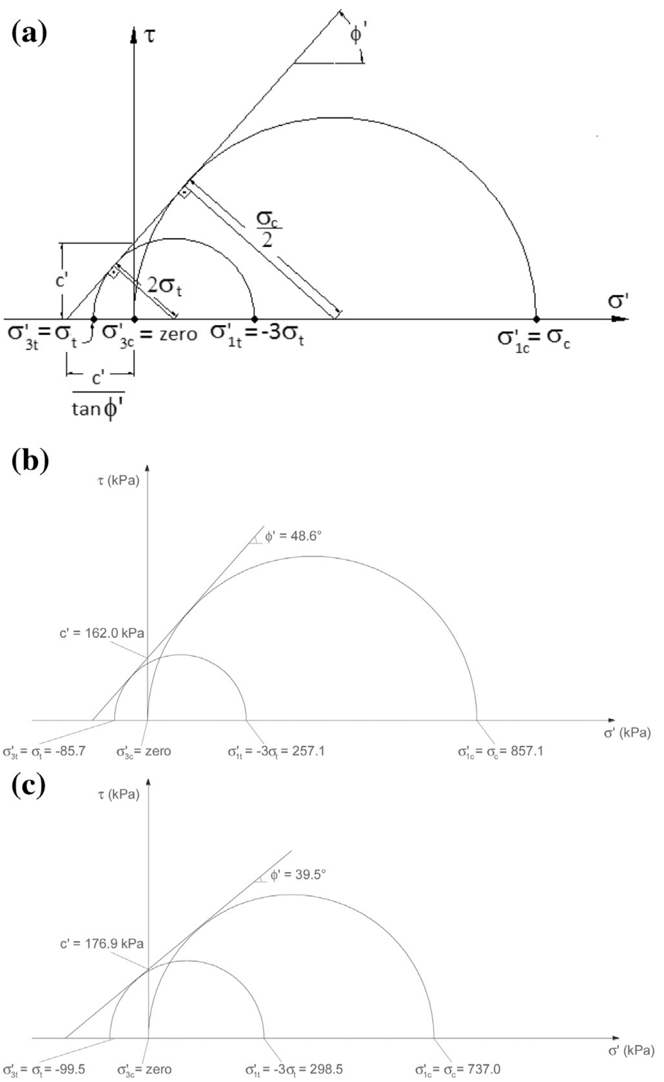

Fig. 1. Mohr–Coulomb envelope based on Mohr circles from splitting tensile and unconfined compression tests: (a) theoretical, (b) real data for fiber-reinforced cemented (3% cement) soil and (c) real data for non-reinforced cemented (3% cement) soil.

principal stress (σ_3') and maximum effective principal stress (σ_1') are $\sigma_{3c}' = $ zero and $\sigma_{3c}' = \sigma_c$ for unconfined compression and $\sigma_{3t}' = \sigma_t$ and $\sigma_{1t}' = -3\sigma_t$ (Jaeger et al., 2007) for splitting tensile tests, it is possible to establish the following equations, based on triangle–rectangle shown in Fig. 1a, respectively for unconfined compression [Eq. (2)] and splitting tensile [Eq. (3)] test results.

$$\sin\phi' = \frac{\frac{\sigma_c}{2}}{\left(\frac{\sigma_c}{2} + \frac{c'}{\tan\phi'}\right)} \tag{2}$$

$$\sin\phi' = \frac{2\sigma_t}{\left(\sigma_t + \frac{c'}{\tan\phi'}\right)} \tag{3}$$

Substituting $[c'/(\tan\phi')]$ of Eq. (2) into Eq. (3) and rearranging it in terms of ($\sin\phi'$), ends up in Eq. (4).

$$\sin\phi' = \frac{\sigma_c - 4\sigma_t}{\sigma_c - 2\sigma_t} \tag{4}$$

In the development of a rational dosage methodology for soil–Portland cement Consoli et al. (2010, 2013) have shown that the porosity/cement ratio (η/C_{iv}), defined as the porosity of the compacted mixture divided by the volumetric cement content, is an appropriate parameter to evaluate the unconfined compressive strength (σ_c) and the splitting tensile strength (σ_t) of Osorio sand–cement and fiber-reinforced clayey sand–cement mixtures, considering the whole range of cement content and the porosity studied. The σ_t/σ_c ratio was shown to be a scalar for the sand–cement and fiber-reinforced clayey sand–cement mixtures studied, being independent of porosity/cement ratio. As a consequence, dosage methodologies based on rational criteria can concentrate either on tensile or compression tests, once they are interdependable. Further studies by Consoli et al. (2012c) have corroborated that the σ_t/σ_c ratio was also a scalar for other soils and cementing agents, such as silt–lime blends. Considering such findings, it is proposed herein to consider that $\sigma_t = \xi\sigma_c$, where ξ is a scalar be introduced into Eqs. (3) and (4), ending in ϕ' and c' being given by Eqs. (5) and (6).

$$\phi' = \arcsin\left(\frac{1-4\xi}{1-2\xi}\right) \tag{5}$$

$$c' = \frac{\sigma_c\left[1 - \left(\frac{1-4\xi}{1-2\xi}\right)\right]}{2\cos\left[\arcsin\left(\frac{1-4\xi}{1-2\xi}\right)\right]} \tag{6}$$

As a consequence, it can be observed that for a given soil, fiber and cementing agent, ξ is a scalar and the effective angle of shearing resistance (ϕ') [given by Eq. (5)] is a constant and consequently is independent of the unconfined compressive strength (σ_c) and the splitting tensile strength (σ_t), as well as of the cement content, porosity or porosity/cement ratio of the studied blend, being a function only of the σ_t/σ_c ratio. On the other side, the effective cohesion intercept (c') of the blend is a function of ξ and σ_c, the latter being a function of porosity/cement ratio (η/C_{iv}). Consequently, c' is a function of the ξ, η and C_{iv}.

3. Checking the proposed methodology

In order to check the accuracy of the methodology presented herein, it will be applied to experimental results carried out by Consoli et al. (1998, 1999) in a glass fiber-reinforced silty sand (as well as in non-reinforced blends) treated with three distinct amounts of ordinary Portland cement (base soil was kept the same throughout the whole experiments).

3.1. Fiber-reinforced silty sand treated with Portland cement

Ulbrich (1997) and Consoli et al. (1998, 1999) carried out complementary studies on the mechanical behavior of fiber-reinforced silty sand treated with Portland cement. Ulbrich (1997) carried out unconfined compressive strength (σ_c) and splitting tensile strength (σ_t) of fiber-reinforced silty sand–cement blends [repeatability of the data was assessed by using three (3) specimens, for each specific cement content], maintaining constant several factors throughout the experiment [fiber percentage of 3% (by weight of dry soil), specimens porosity of about 33.8%, curing period of 7 days and degree of saturation above 95%]. The silty sand had 1%, 3% and 5% (by weight of dry soil plus fiber) of ordinary Portland cement content added to it, resulting in unconfined compressive strengths (σ_c) of 449 kPa, 857 kPa and 1134 kPa [average values of three (3) specimens for each cement content], respectively, being characterized as weak to strongly cemented geomaterials. The relation splitting tensile strength (σ_t) to unconfined compressive strength (σ_c) was found to be $\xi = 0.10$. Nine (9) drained triaxial tests under low confining pressures of 20, 60 and 100 kPa were carried out with

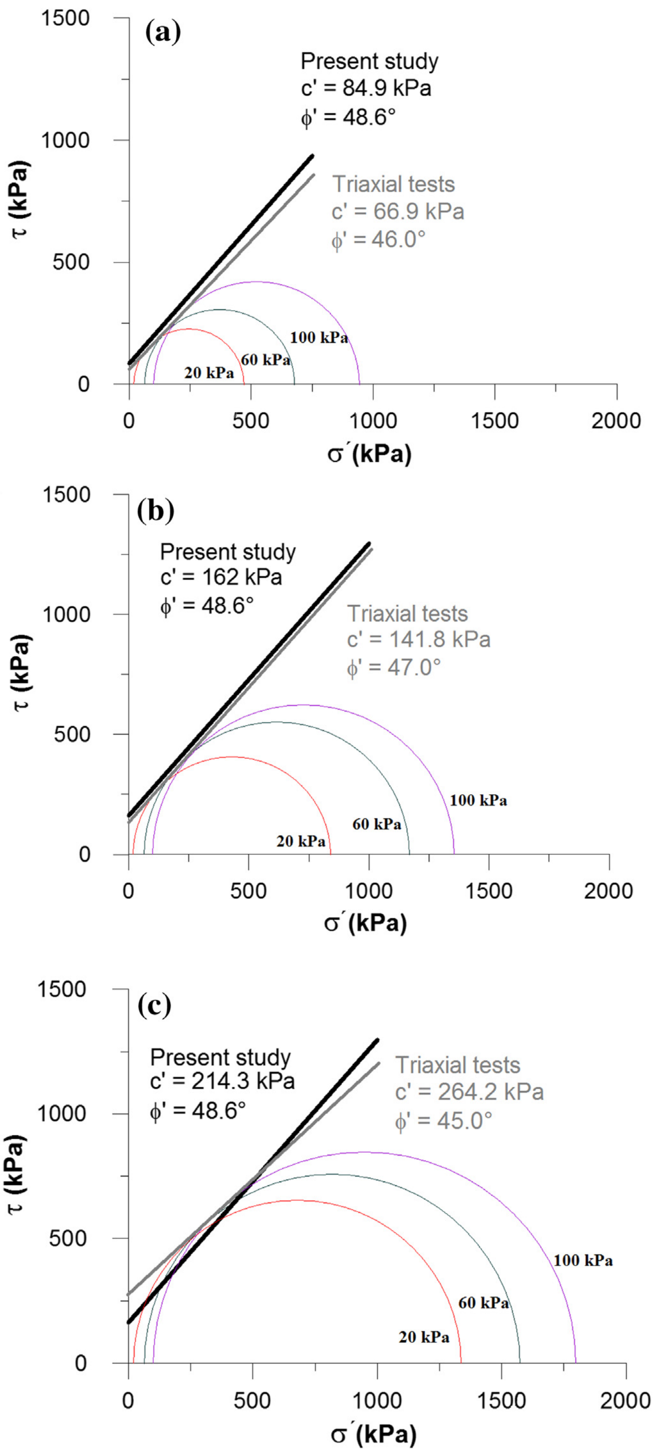

Fig. 2. Fiber-reinforced cemented silty sand Mohr–Coulomb failure envelopes [using methodology developed in present research and results based in the Mohr circles] in τ–σ' stress space for three triaxial specimens considering (a) 1% cement content, (b) 3% cement content and (c) 5% cement content, besides of 3% fiber content and confining pressures varying from 20 to 100 kPa.

amounts of Portland cement varying from 1% to 5% and keeping porosity of the blends about 33.8%.

Inserting ξ value (0.10) into Eqs. (5) and (6), yields in $\phi' = 48.6°$ and $c' = 0.189\sigma_c$.

Unconfined compressive strengths values end up in the effective cohesion intercept (c') of 84.9 kPa, 162 kPa and 214.3 kPa, respectively for 1%, 3% and 5% cement content, all containing 3% glass fiber content. Typical estimative of failure envelope obtained using methodology introduced in present study is presented in Fig. 2b containing 3% glass fiber and 3% cement content.

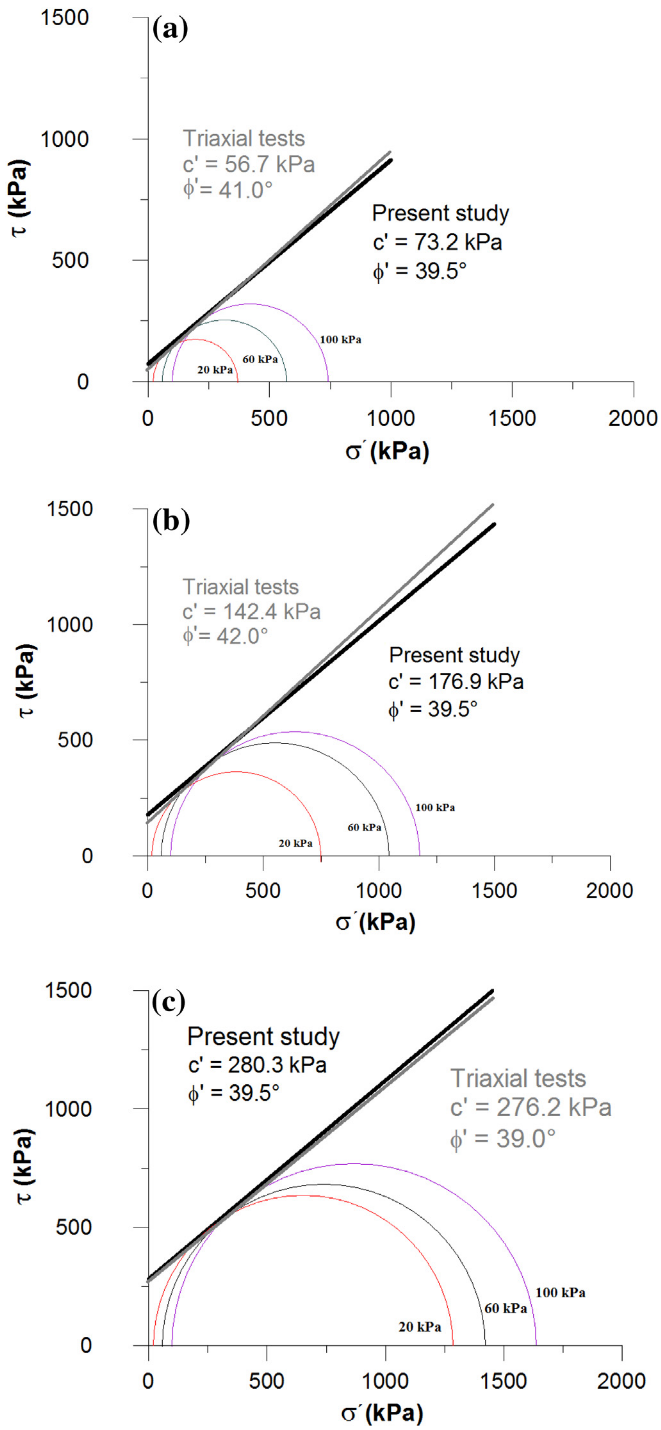

Fig. 3. Cemented silty sand Mohr–Coulomb failure envelopes [using methodology developed in present research and results based in the Mohr circles] in τ–σ' stress space for three triaxial specimens considering (a) 1% cement content, (b) 3% cement content and (c) 5% cement content and confining pressures varying from 20 to 100 kPa.

Fig. 2a–c presents the Mohr semi-circles and respective failure envelopes of fiber-reinforced cemented sand triaxial peak shear strength in a τ–σ' stress space (considering the three confining pressures used in the research – 20, 60 and 100 kPa) respectively for 1% cement content, 3% cement content and 5% cement content,

Table 1
Comparison of ϕ' and c' results for fiber-reinforced and non-reinforced cemented silty sand specimens (considering cement contents of 1%, 3% and 5%), established after methodology developed herein (based on σ_t/σ_c ratio is unique for a given sand–cement blend or fiber-reinforced sand–cement mix), as well as based on triaxial tests (Consoli et al., 1999).

Cement content (%)	Non-reinforced cemented silty sand				Fiber-reinforced cemented silty sand			
	c' (kPa) [triaxial tests]	c' (kPa) [present study]	ϕ' (°) [triaxial tests]	ϕ' (°) [present study]	c' (kPa) [triaxial tests]	c' (kPa) [present study]	ϕ' (°) [triaxial tests]	ϕ' (°) [present study]
1	56.7	73.2	41.0	39.5	66.9	84.9	46.0	48.6
3	142.4	176.9	42.0	39.5	141.8	162.0	47.0	48.6
5	276.2	280.3	39.0	39.5	264.2	214.3	45.0	48.6

as well as the Mohr–Coulomb failure envelopes whose parameters were estimated above using methodology developed in present research. It can be observed in Fig. 2a–c that the Mohr–Coulomb failure envelopes drawn using $\phi' = 48.6°$ (for all cement contents) and $c' = 84.9$ kPa (1% cement), 162 kPa (3% cement) and 214.3 kPa (5% cement) [values obtained based on the methodology developed herein – using unconfined compression and splitting tensile tests, whose equipment can be found even under minimal laboratory facilities] are a sound representation of the tangent to the Mohr semi-circles drawn based on triaxial testing, at distinct effective confining stresses, of the studied fiber-reinforced silty sand–cement blends.

One point that must be recalled is that in order to have representativity, saturated drained triaxial failure envelope parameters will be effectively represented by splitting tensile and unconfined compression tests carried out in specimens having high degrees of saturation, above 90%, in which suctions are nearly zero. Such conditions are usually reached in fiber-reinforced cemented sandy soils by submerging the specimens in water for at least 24 h before testing (Consoli et al., 2013). Another way would be to carry out some measurements of suction directly in the specimens.

3.2. Silty sand treated with Portland cement

Consoli et al. (1996, 1998, 1999) carried out complementary studies on the mechanical behavior of a silty sand derived from Botucatu weathered sandstone treated with cement. Several factors were maintained constant throughout the experiment (void ratio of 0.51, curing period of 7 days and degree of saturation above 95%). The silty sand had 1%, 3% and 5% (by weight of dry soil) of ordinary cement content added to it, having unconfined compressive strengths (σ_c) of 305.0 kPa, 737.0 kPa and 1168.0 kPa [average values of three (3) specimens for each cement content], respectively, being characterized as weak to strongly cemented soils. The relation splitting tensile strength (σ_t) to unconfined compressive strength (σ_c) was found to be $\xi = 0.135$. Nine (9) drained triaxial tests under low confining pressures of 20, 60 and 100 kPa were carried out with amounts of Portland cement varying from 1% to 5% and porosity of the blend of about 33.8%.

Inserting ξ value (0.135) into Eqs. (5) and (6), yields in $\phi' = 39.5°$ and $c' = 0.24\sigma_c$.

Inserting the unconfined compressive strengths values into Eq. (6) ends up that the effective cohesion intercept (c') are respectively 73.2 kPa, 176.9 kPa and 280.3 kPa, respectively for 1%, 3% and 5% cement content. Typical estimative of failure envelope obtained using methodology introduced in present study is presented in Fig. 2c containing 3% cement content.

Fig. 3a–c presents the Mohr semi-circles and respective failure envelopes of triaxial peak shear strength in a τ–σ stress space (considering the three confining pressures used in the research – 20, 60 and 100 kPa) respectively for 1%, 3% and 5% cement content, as well as the Mohr–Coulomb failure envelopes whose parameters were calculated above using methodology developed in present research. It can be observed in Fig. 3a–c that the Mohr–Coulomb failure envelopes drawn using $\phi' = 39.5°$ (for all cement contents) and $c' = 73.2$ kPa (1% cement), 176.9 kPa (3% cement) and 280.3 kPa (5% cement) are an all-encompassing representation of the tangent to the Mohr semi-circles drawn based on triaxial testing, at low effective confining pressures, of the studied silty sand–cement blends.

A comparison of ϕ' and c' results of both fiber-reinforced and non-reinforced cemented silty sand specimens (considering cement contents of 1, 3 and 5%), established after methodology developed herein, as well as based on triaxial tests (Consoli et al., 1999) is shown in Table 1. It can be observed that the effective angles of shear resistance (ϕ') calculated after methodology developed herein for non-reinforced (39.5°) and fiber-reinforced (48.6°) cemented sand are a fairly good representation of ϕ' obtained directly after analyzing triaxial test results on non-reinforced (on average 41°) and fiber-reinforced (on average 46°) cemented sand. Regarding the effective cohesion intercept (c') calculated after methodology developed herein for non-reinforced (varying from 73.2 to 280.3 kPa) and fiber-reinforced (varying from 84.9 to 214.3 kPa) cemented sand is a reasonably good representation of ϕ' obtained directly after analyzing triaxial test results on non-reinforced (varying from 56.7 to 276.2 kPa) and fiber-reinforced (varying from 66.9 to 264.2 kPa) cemented sand.

4. Conclusions

From the data presented in this manuscript the following conclusions can be drawn:

- A methodology for estimating Mohr–Coulomb failure envelope parameters based on splitting tensile strength (σ_t) and unconfined compressive strength (σ_c) of both fiber-reinforced and non-reinforced artificially cemented sandy soils is proposed. The proposed methodology was shown to be successful regarding determination of the effective angle of shear resistance (ϕ') and the effective cohesion intercept (c') for both fiber-reinforced and non-reinforced cemented sandy soils.
- Present study is limited to low confining pressures (up to 100 kPa), because at higher stresses the failure envelope must curve and the method would become unconservative.
- Other limitations are linked to soil, fiber and cement studied herein and further studies are still necessary to check if such methodology might be spread to other soils (e.g., clays), fibers (e.g., polypropylene, polyester, nylon), as well as to other cement agents, such as lime, fly ash–lime, etc.

Acknowledgments

The authors wish to express their gratitude to Brazilian MCT/CNPq (projects Produtividade em Pesquisa, Iniciação Científica, Edital Universal and INCT–REAGEO) for the financial support to the research group.

References

Clough, G.W., Sitar, N., Bachus, R.C., Rad, N.S., 1981. Cemented sands under static loading. Journal of the Geotechnical Engineering Division 107 (6), 799–817. ASCE.

Consoli, N.C., Schnaid, F., Rohlfes Jr., J.A., Prietto, P.D.M., 1996. Engineering properties of residual soil–cement mixtures. In: Yonekura, R., Terashi, M., Shibazaki, M. (Eds.), Proceedings of the Second International Conference on Ground Improvement Geosystems, Tokyo, Grouting and Deep Mixing, vol. 1, ISBN 90 5410 805 3. Rotterdam.

Consoli, N.C., Prietto, P.D.M., Ulbrich, L.A., 1998. Influence of fiber and cement addition on behavior of sandy soil. Journal of Geotechnical and Geoenvironmental Engineering 124 (12), 1211–1214. ASCE.

Consoli, N.C., Prietto, P.D.M., Ulbrich, L.A., 1999. The behaviour of a fibre-reinforced cemented soil. Ground Improvement 3 (1), 21–30. Thomas Telford.

Consoli, N.C., Vendruscolo, M.A., Prietto, P.D.M., 2003. Behavior of plate load tests on soil layers improved with cement and fiber. Journal of Geotechnical and Geoenvironmental Engineering 129 (1), 96–101. ASCE.

Consoli, N.C., Foppa, D., Festugato, L., Heineck, K.S., 2007. Key parameters for strength control of artificially cemented soils. Journal of Geotechnical and Geoenvironmental Engineering 133 (2), 197–205. ASCE.

Consoli, N.C., Vendruscolo, M.A., Fonini, A., Dalla Rosa, F., 2009. Fiber reinforcement effects on sand considering a wide cementation range. Geotextiles and Geomembranes 27 (3), 196–203.

Consoli, N.C., Cruz, R.C., Floss, M.F., Festugato, L., 2010. Parameters controlling tensile and compressive strength of artificially cemented sand. Journal of Geotechnical and Geoenvironmental Engineering 136 (5), 759–763. ASCE.

Consoli, N.C., Cruz, R.C., Viana da Fonseca, A., Coop, M.R., 2012a. Influence of cement–voids ratio on stress–dilatancy behavior of artificially cemented sand. Journal of Geotechnical and Geoenvironmental Engineering 138 (1), 100–109. ASCE.

Consoli, N.C., Thomé, A., Girardello, V., Ruver, C., 2012b. Uplift behavior of plates embedded in fiber-reinforced cement stabilized backfill. Geotextiles & Geomembranes 35 (12), 107–111.

Consoli, N.C., Johann, A.D.R., Gauer, E.A., Santos, V.R., Moretto, R.L., Corte, M.B., 2012c. Key parameters for tensile and compressive strength of silt–lime mixtures. Géotechnique Letters 2 (3), 81–85.

Consoli, N.C., Moraes, R.R., Festugato, L., 2013. Parameters controlling tensile and compressive strength of fiber-reinforced cemented soil. Journal of Materials in Civil Engineering. http://dx.doi.org/10.1061/(ASCE)MT.1943-5533.0000555. ASCE.

Dalla Rosa, F., Consoli, N.C., Baudet, B.A., 2008. An experimental investigation of the behavior of artificially cemented soil cured under stress. Géotechnique 58 (8), 675–679.

Festugato, L., Fourie, A., Consoli, N.C., 2013. Cyclic shear response of fibre-reinforced cemented paste backfill. Géotechnique Letters 3 (1), 5–12. http://dx.doi.org/10.1680/geolett.12.00042.

Jaeger, J.C., Cook, N.G.W., Zimmerman, R.W., 2007. Fundamentals of Rock Mechanics, fourth ed. Blackwell Publishing, p. 475.

Ulbrich, L.A., 1997. Macroscopic and Microscopic Behaviour of a Fibre-reinforced Cemented Soils. M.Sc. Thesis. Federal University of Rio Grande do Sul (in Portuguese).

Notation

c': effective cohesion intercept
C_{iv}: volumetric cement content
F: fiber content
ξ: σ_t/σ_c ratio
ϕ': effective angle of shearing resistance
η: porosity
σ': effective normal stress
σ_t: splitting tensile strength
σ_c: unconfined compressive strength
σ_1': maximum effective principal stress
σ_3': minimum effective principal stress
τ: shear stress