Technical note

A practical methodology for the determination of failure envelopes of fiber-reinforced cemented sands

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1. Introduction

Determination of Mohr–Coulomb failure envelope parameters of fiber-reinforced/non-reinforced artificially cemented soils requires carrying out triaxial tests (e.g., Clough et al., 1981; Consoli et al., 2007, 2009, 2012a, 2013; Dalla Rosa et al., 2008), simple shear (Festugato et al., 2013), amongst many other complex and time consuming tests.

An alternative methodology to estimate Mohr–Coulomb failure envelope parameters of fiber-reinforced/non-reinforced artificially cemented soils is suggested in present work. The concept is to carry out basic tests, such as unconfined compression and splitting tensile tests, whose equipment (loading machine and proving rings) can be found even under slight laboratory facilities. Besides, the methodology to be presented herein intends to allow increasing reliability and widening range of validity of the results, once the setup of basic (splitting tensile and unconfined compression) tests carried out for a given sandy soil and a specific cement agent will allow determining $c'$ and $\phi'$ for any specific condition comprised inside the range of porosity and amount of cement employed during basic testing. Types of applications could fit improvement behavior of shallow foundations bearing on soil layers enhanced with cement and fiber (Consoli et al., 2003) and enhanced uplift performance of anchor plates embedded in fiber-reinforced cement stabilized backfill (Consoli et al., 2012b). During design considerations, once major difficulties usually occur during mixture procedures, the precision obtained using the methodology proposed to estimate Mohr–Coulomb failure envelope parameters of fiber-reinforced/non-reinforced artificially cemented materials is usually good.

2. Mohr–Coulomb failure theory

The Mohr–Coulomb failure theory is represented in the shear strength ($\tau$) versus effective normal stress ($\sigma'$) space by plotting Mohr semi-circles representing stress states at failure and then drawing a tangent to these semi-circles, which represents the Mohr–Coulomb failure envelope. As presented in Fig. 1a, in the Mohr–Coulomb failure theory, the shear strength ($\tau$) of a given material is assumed, considering effective stress conditions, to vary linearly with effective normal stress ($\sigma'$), according to two parameters: effective cohesion intercept ($c'$) and effective angle of shearing resistance ($\phi'$), as shown in Eq. (1).

$$\tau = c' + \sigma' \tan \phi'$$

(1)

Using unconfined compression and splitting tensile tests principal stress states at failure, in which, the minimum effective
In terms of \((\sin \phi)\) and maximum effective principal stress \((\sigma_1)\) are \(\sigma_3 = \sigma_c\) for unconfined compression and \(\sigma_3 = \sigma_t\) and \(\sigma_1 = -3\sigma_t\) (Jaeger et al., 2007) for splitting tensile tests, it is possible to establish the following equations, based on triangle–rectangle shown in Fig. 1a, respectively for unconfined compression [Eq. (2)] and splitting tensile [Eq. (3)] test results.

\[
\sin \phi' = \frac{\sigma_t}{(\sigma_1 + \sigma_t)} \tan \phi
\]

\[
\sin \phi' = \frac{2\sigma_t}{(\sigma_1 + \sigma_t)}
\]

Substituting \([c'/(\tan \phi')\)] of Eq. (2) into Eq. (3) and rearranging it in terms of \((\sin \phi')\), ends up in Eq. (4).

\[
\sin \phi' = \frac{\sigma_c - 4\sigma_t}{\sigma_c - 2\sigma_t}
\]

In the development of a rational dosage methodology for soil–Portland cement Consoli et al. (2010, 2013) have shown that the porosity/cement ratio \((\eta/C_w)\), defined as the porosity of the compacted mixture divided by the volumetric cement content, is an appropriate parameter to evaluate the unconfined compressive strength \((\sigma_c)\) and the splitting tensile strength \((\sigma_t)\) of Osorio sand–cement and fiber-reinforced clayey sand–cement mixtures, considering the whole range of cement content and the porosity studied. The \(\sigma_t/\sigma_c\) ratio was shown to be a scalar for the sand–cement and fiber-reinforced clayey sand–cement mixtures studied, being independent of porosity/cement ratio. As a consequence, dosage methodologies based on rational criteria can concentrate either on tensile or compression tests, once they are interdependable. Further studies by Consoli et al. (2012c) have corroborated that the \(\sigma_t/\sigma_c\) ratio was also a scalar for other soils and cementing agents, such as silt–lime blends. Considering such findings, it is proposed herein to consider that \(\sigma_t = \xi \sigma_c\), where \(\xi\) is a scalar being independent of the porosity/cement ratio. As a consequence, it can be observed that for a given soil, fiber and cementing agent, \(\xi\) is a scalar and the effective angle of shearing resistance \((\phi')\) [given by Eq. (5)] is a constant and consequently is independent of the unconfined compressive strength \((\sigma_c)\) and the splitting tensile strength \((\sigma_t)\), as well as of the cement content, porosity or porosity/cement ratio of the studied blend, being a function only of the \(\sigma_t/\sigma_c\) ratio. On the other side, the effective cohesion intercept \((c')\) of the blend is a function of \(\xi\) and \(\sigma_c\), the latter being a function of porosity/cement ratio \((\eta/C_w)\). Consequently, \(c'\) is a function of the \(\xi, \eta, \) and \(C_w\).

3. Checking the proposed methodology

In order to check the accuracy of the methodology presented herein, it will be applied to experimental results carried out by Consoli et al. (1998, 1999) in a glass fiber-reinforced silty sand (as well as in non-reinforced blends) treated with three distinct amounts of ordinary Portland cement (base soil was kept the same throughout the whole experiments).

3.1. Fiber-reinforced silty sand treated with Portland cement

Ulbrich (1997) and Consoli et al. (1998, 1999) carried out complementary studies on the mechanical behavior of fiber-reinforced silty sand treated with Portland cement. Ulbrich (1997) carried out unconfined compressive strength \((\sigma_c)\) and splitting tensile strength \((\sigma_t)\) of fiber-reinforced silty sand–cement blends [repeatability of the data was assessed by using three (3) specimens, for each specific cement content], maintaining constant several factors throughout the experiment [fiber percentage of 3% by weight of dry soil], specimens porosity of about 33.8%, curing period of 7 days and degree of saturation above 95%. The silty sand had 1%, 3% and 5% (by weight of dry soil plus fiber) of ordinary Portland cement content added to it, resulting in unconfined compressive strengths \((\sigma_c)\) of 449 kPa, 857 kPa and 1134 kPa [average values of three (3) specimens for each cement content], respectively, being characterized as weak to strongly cemented geomaterials. The relation splitting tensile strength \((\sigma_t)\) to unconfined compressive strength \((\sigma_c)\) found to be \(\xi = 0.10\). Nine (9) drained triaxial tests under low confining pressures of 20, 60 and 100 kPa were carried out with

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amounts of Portland cement varying from 1% to 5% and keeping porosity of the blends about 33.8%.

Inserting $x$ value (0.10) into Eqs. (5) and (6), yields $f_0 = 48.6$ and $c_0 = 0.189$.

Unconfined compressive strengths values end up in the effective cohesion intercept ($c^\prime$) of 84.9 kPa, 162 kPa and 214.3 kPa, respectively for 1%, 3% and 5% cement content, all containing 3% fiber content and confining pressures varying from 20 to 100 kPa.

Fig. 2. Fiber-reinforced cemented silty sand Mohr–Coulomb failure envelopes [using methodology developed in present research and results based in the Mohr circles] in $\tau$–$\sigma$ stress space for three triaxial specimens considering (a) 1% cement content, (b) 3% cement content and (c) 5% cement content, besides of 3% fiber content and confining pressures varying from 20 to 100 kPa.

Fig. 3. Cemented silty sand Mohr–Coulomb failure envelopes [using methodology developed in present research and results based in the Mohr circles] in $\tau$–$\sigma$ stress space for three triaxial specimens considering (a) 1% cement content, (b) 3% cement content and (c) 5% cement content and confining pressures varying from 20 to 100 kPa.

Typical estimative of failure envelope obtained using methodology introduced in present study is presented in Fig. 2b containing 3% glass fiber and 3% cement content.

Fig. 2a–c presents the Mohr semi-circles and respective failure envelopes of fiber-reinforced cemented sand triaxial peak shear strength in a $\tau$–$\sigma$ stress space (considering the three confining pressures used in the research – 20, 60 and 100 kPa) respectively for 1% cement content, 3% cement content and 5% cement content,

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as well as the Mohr–Coulomb failure envelopes whose parameters were estimated above using methodology developed in present research. It can be observed in Fig. 2a–c that the Mohr–Coulomb failure envelopes drawn using \( \phi' = 48.6^\circ \) (for all cement contents) and \( c' = 84.9 \) kPa (1% cement), 162 kPa (3% cement) and 214.3 kPa (5% cement) [values obtained based on the methodology developed herein – using unconfined compression and splitting tensile tests, whose equipment can be found even under minimal laboratory facilities] are a sound representation of the tangent to the Mohr semi-circles drawn based on triaxial testing, at distinct effective confining stresses, of the studied fiber-reinforced silty sand–cement blends.

One point that must be recalled is that in order to have representativeness, saturated drained triaxial failure envelope parameters will be effectively represented by splitting tensile and unconfined compression tests carried out in specimens having high degrees of saturation, above 90%, in which suctions are nearly zero. Such conditions are usually reached in fiber-reinforced cemented sandy soils by submerging the specimens in water for at least 24 h before testing (Consoli et al., 2013). Another way would be to carry out some measurements of suction directly in the specimens.

### 3.2. Silty sand treated with Portland cement

Consoli et al. (1996, 1998, 1999) carried out complementary studies on the mechanical behavior of a silty sand derived from Botucatu weathered sandstone treated with cement. Several factors were maintained constant throughout the experiment (void ratio of 0.51, curing period of 7 days and degree of saturation above 95%). The silty sand had 1%, 3% and 5% (by weight of dry soil) of ordinary cement content added to it, having unconfined compressive strengths \( (\sigma_c) \) of 305.0 kPa, 737.0 kPa and 1168.0 kPa [average values of three (3) specimens for each cement content], respectively, being characterized as weak to strongly cemented soils. The relation splitting tensile strength \( (\sigma_t) \) to unconfined compressive strength \( (\sigma_c) \) was found to be \( t = 0.135 \). Nine (9) drained triaxial tests under low confining pressures of 20, 60 and 100 kPa were carried out with amounts of Portland cement varying from 1% to 5% and porosity of the blend of about 33.8%.

Inserting \( t \) value (0.135) into Eqs. (5) and (6), yields in \( \phi' = 39.5^\circ \) and \( c' = 0.24\sigma_c \).

Inserting the unconfined compressive strengths values into Eq. (6) ends up that the effective cohesion intercept \( (c') \) are respectively 73.2 kPa, 176.9 kPa and 280.3 kPa, respectively for 1%, 3% and 5% cement content. Typical estimative of failure envelope obtained using methodology introduced in present study is presented in Fig. 2c containing 3% cement content.

Fig. 3a–c presents the Mohr semi-circles and respective failure envelopes of triaxial peak shear strength in a \( t-c' \) stress space (considering the three confining pressures used in the research – 20, 60 and 100 kPa) respectively for 1%, 3% and 5% cement content, as well as the Mohr–Coulomb failure envelopes whose parameters were calculated above using methodology developed in present research. It can be observed in Fig. 3a–c that the Mohr–Coulomb failure envelopes drawn using \( \phi' = 39.5^\circ \) (for all cement contents) and \( c' = 73.2 \) kPa (1% cement), 176.9 kPa (3% cement) and 280.3 kPa (5% cement) are an all-encompassing representation of the tangent to the Mohr semi-circles drawn based on triaxial testing, at low effective confining pressures, of the studied silty sand–cement blends.

A comparison of \( \phi' \) and \( c' \) results of both fiber-reinforced and non-reinforced cemented silty sand specimens (considering cement contents of 1, 3 and 5%), established after methodology developed herein, as well as based on triaxial tests (Consoli et al., 1999) is shown in Table 1. It can be observed that the effective angles of shear resistance \( (\phi') \) calculated after methodology developed herein for non-reinforced (39.5°) and fiber-reinforced (48.6°) cemented sand are a fairly good representation of \( \phi' \) obtained directly after analyzing triaxial test results on non-reinforced (on average 41°) and fiber-reinforced (on average 46°) cemented sand. Regarding the effective cohesion intercept \( (c') \) calculated after methodology developed herein for non-reinforced (varying from 73.2 to 280.3 kPa) and fiber-reinforced (varying from 84.9 to 214.3 kPa) cemented sand is a reasonably good representation of \( c' \) obtained directly after analyzing triaxial test results on non-reinforced (varying from 56.7 to 276.2 kPa) and fiber-reinforced (varying from 66.9 to 264.2 kPa) cemented sand.

### 4. Conclusions

From the data presented in this manuscript the following conclusions can be drawn:

- A methodology for estimating Mohr–Coulomb failure envelope parameters based on splitting tensile strength \( (\sigma_t) \) and unconfined compressive strength \( (\sigma_c) \) of both fiber-reinforced and non-reinforced artificially cemented sandy soils is proposed. The proposed methodology was shown to be successful regarding determination of the effective angle of shear resistance \( (\phi') \) and the effective cohesion intercept \( (c') \) for both fiber-reinforced and non-reinforced cemented sandy soils.

- Present study is limited to low confining pressures (up to 100 kPa), because at higher stresses the failure envelope must curve and the method would become unconservative.

- Other limitations are linked to soil, fiber and cement studied herein and further studies are still necessary to check if such methodology might be spread to other soils (e.g., clays), fibers (e.g., polypropylene, polyester, nylon), as well as to other cement agents, such as lime, fly ash–lime, etc.

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References


Notation

$c'$: effective cohesion intercept
$c_v$: volumetric cement content
$F$: fiber content
$f_0$: ratio
$\gamma'$: effective angle of shear resistance
$\eta$: porosity
$s'$: effective normal stress
$s_1$: splitting tensile strength
$s_2$: unconfined compressive strength
$s_c$: maximum effective principal stress
$s_t$: minimum effective principal stress
$t$: shear stress